FM
Frequency Modulation

Let $m(t)$: message signal at baseband ($f\downarrow$)
$c(t)$: Carrier signal at Passband ($f\uparrow$)

where:-

$$C(t) = A_c \cdot \cos(\theta_i(t))$$
$$c(t) = A_c \cos(2\pi f_c \cdot t)$$

$\boxed{A_c}$ Carrier Amplitude
if $m(t)$ modulates the amplitude $\rightarrow$ AM

$\boxed{\theta_i(t)}$ Carrier angle
if $m(t)$ modulates the angle
$\rightarrow$ Angle Modulation

- **Angle Modulation:-**

In angle modulation, the phase angle of the carrier $\theta_i(t)$ is varied according to the message signal $m(t)$. not the amplitude as in AM.

Angle modulation

① Phase modulation (PM)

تتغير $\theta_i(t)$ بشكل مباشر مع ال $m(t)$.

$$\theta_{i_{Pm}}(t) = 2\pi f_c t + k_p \cdot m(t)$$

k_p: phase modulation Sensitivity

PM equation
$$S_{PM}(t) = A_c \cos(2\pi f_c t + k_p \cdot m(t))$$
Carrier after modulation

② Frequency Modulation FM

حيث يتغير تردد ال C(t) بشكل مباشر مع ال m(t)

$$f_c' = f_c + K_f \cdot m(t)$$

↳ التردد بعد التعديل

$$\because \theta = \int \omega \cdot dt$$

$$\theta_i(t) = 2\pi \int f_c' \, dt = 2\pi f_c t + 2\pi K_f \int m(t)\, dt$$

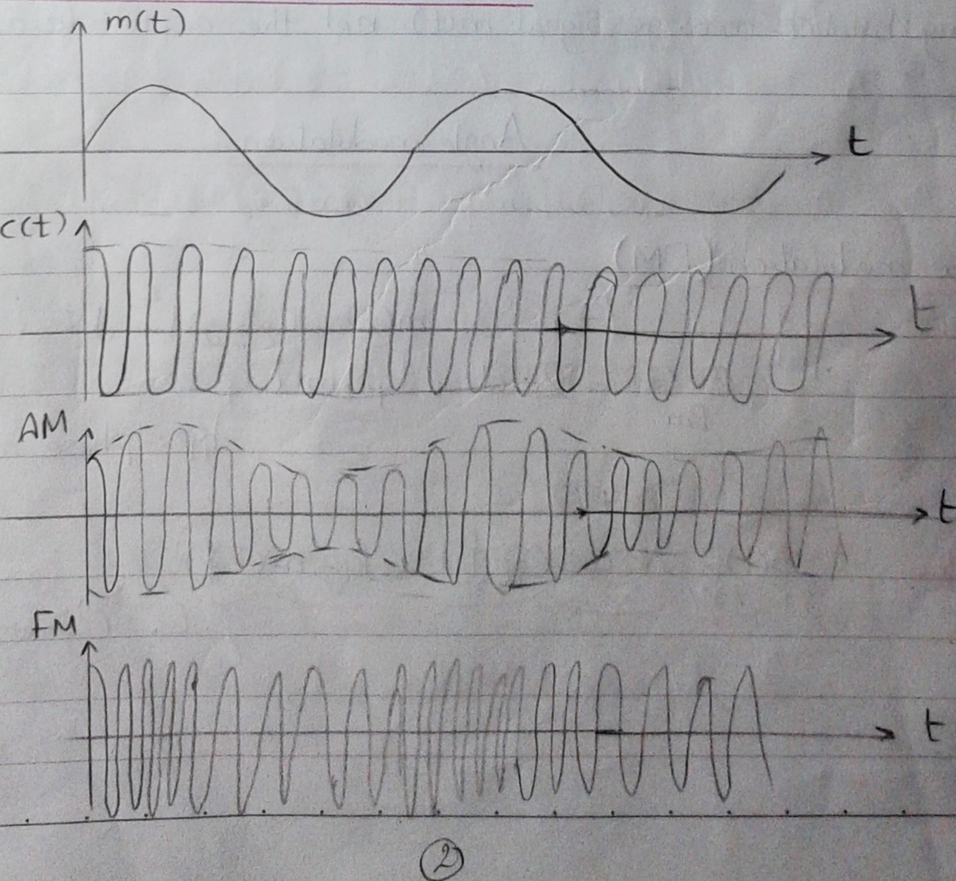
$$\boxed{S_{FM}(t) = A_c \cos\left(2\pi f_c t + 2\pi K_f \int m(t)\, dt\right)}$$

→ FM equation

$f_i(t) = f_c + K_f \cdot m(t)$ — K_f: Frequency modulation Sensitivity

↳ instantenous frequency

Time Representation of modulated Signals :-



عند مقارنة ال FM بال PM نلاحظ أنه يمكن توليد ال FM من ال PM والعكس صحيح

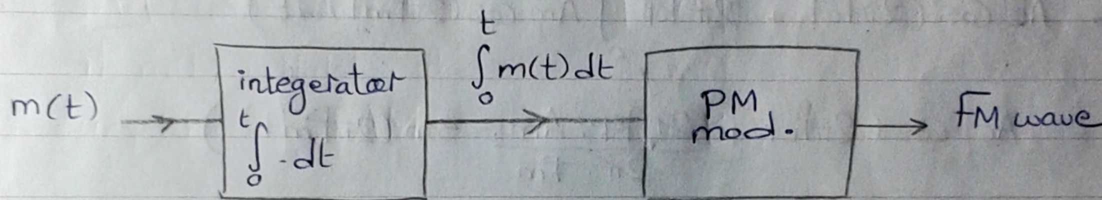
Q: How to generate FM from PM & PM from FM ??

Sol.

$$\because \; S(t)_{PM} = A_c \cos(2\pi f_c t + k_p m(t))$$

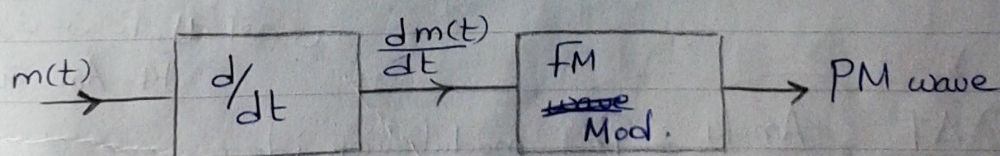
$$S(t)_{FM} = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right)$$

ⓐ FM from PM



وضعنا integerator بعد m(t) لكي نحصل على $\int_0^t m(t)$

ⓑ PM from FM



وضعنا d/dt بعد ال m(t) لكي نحصل على تفاضل ال m(t) كما بالرسم.

Single Tone FM modulation

① Narrow Band FM — N.B.F.M

② Wide Band FM — W.B.F.M

let $m(t) = A_m \cos 2\pi f_m t$, B.W of $m(t) = f_m$

$c(t) = A_c \cos 2\pi f_c t$, $f_c \gg f_m$

$$S_{fm}(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int m(t)\,dt\right)$$

$$S_{fm}(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t A_m \cos(2\pi f_m t)\,dt\right)$$

$$= A_c \cos\left(2\pi f_c t + \frac{\cancel{2\pi} k_f . A_m}{\cancel{2\pi} . f_m} \sin(2\pi f_m t)\right)$$

let $\Delta f = k_f . A_m$ max. freq. deviation

أقصى انحراف عن f_c

حفظ

$\therefore f_c' = f_c + k_f\, m(t)$

الانحراف عن f_c وله أقصى

قيمة $k_f . A_m$

حفظ

let $\dfrac{\Delta f}{f_m} = \dfrac{A_m . k_f}{f_m} = \beta$ modulation idex

بالتعويض عن β في المعادلة الأخيرة

حفظ وهام جداً

$$S_{FM}(t) = A_c \cos\left(2\pi f_c t + \beta \sin(2\pi f_m t)\right)$$

معادلة الـ FM العامة

$\beta = \dfrac{\Delta f}{f_m}$ modulation index

if $0 < \beta < 1$
$\therefore$ N.B.F.M

if $1 \leqslant \beta$
$\therefore$ W.B.F.M

ⓐ For N.B.F.M

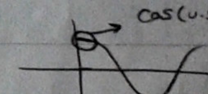
$$S(t)_{FM} = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t)) \quad , \quad 0 < B < 1$$

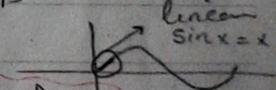
Note that:- $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$S(t)_{FM} = A_c \cos(2\pi f_c t) . \cos(\beta \sin(2\pi f_m t)) - A_c \sin(2\pi f_c t)$$
$$\sin(\beta \sin(2\pi f_m t)) .$$

since $0 < B < 1$ β very small

$$\cos(\beta \sin(2\pi f_m t)) \simeq \cos 0 \cong 1$$
$$\sin(\beta \sin(2\pi f_m t)) \simeq \beta \sin(2\pi f_m t)$$





$$\therefore \underset{N.B.F.M}{S(t)} = \underbrace{A_c \cos(2\pi f_c t)}_{c(t)} - \beta(A_c \sin(2\pi f_c t)).(\sin(2\pi f_m t))$$

الصيغة النهائية لل N.B.F.M

• Spectrum of NBFM

$$\underset{NBFM}{S(t)} = \underbrace{A_c \cos(2\pi f_c t)}_{C(t)} - \underbrace{\beta (A_c \sin(2\pi f_c t))(\sin(2\pi f_m t))}_{D.S.B}$$

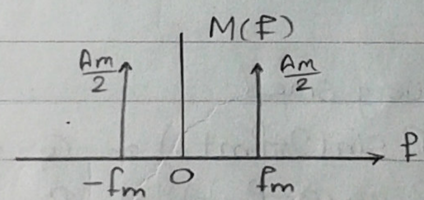
$$= A_c \cos(2\pi f_c t) - \frac{\beta A_c}{2} \cos 2\pi (f_c - f_m) t + \frac{\beta A_c}{2} \cos 2\pi (f_c + f_m) t$$

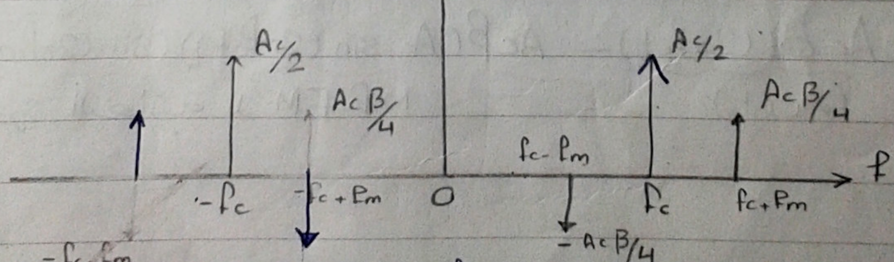
$$= \underset{\downarrow \\ C(t)}{A_c \cos(2\pi f_c t)} + \underset{\downarrow \\ U.S.B.}{\frac{\beta A_c}{2} \cos(2\pi (f_c + f_m) t)} - \underset{\downarrow \\ L.S.B}{\frac{\beta A_c}{2} \cos(2\pi (f_c - f_m) t)}$$

$$S(f) = \frac{A_c}{2} [\delta(f - f_c) + \delta(f + f_c)] + \frac{\beta A_c}{4} [\delta(f - (f_c + f_m)) + \delta(f + (f_c + f_m))]$$

$$- \frac{\beta A_c}{4} [\delta(f + (f_c - f_m)) + \delta(f - (f_c - f_m))].$$

• الشكل التالى يوضح ال NBFM Spectrum





Spectrum of NBFM

هام جداً جداً

NBFM Power

P_{total} = total transmitted Power = P_t

$$P_t = P_c + P_{U.S.B} + P_{L.S.B}$$

* $P_c = \frac{A_c^2}{2}$ * $P_{L.S.B} = P_{U.S.B} = \frac{A_c^2 \beta^2}{8}$

* $P_{D.S.B.} = \frac{A_c^2 \beta^2}{4}$

* $P_t = P_c + P_{DSB} = P_c \left(1 + \frac{\beta^2}{2}\right)$

B.W. of $m(t) = f_m$
B.W. of NBFM $= 2f_m$

حفظ

نلاحظ أن الـ NBFM يشبه الـ AM ولكن أيهما أفضل؟؟

بالرغم من أن الـ NBFM يشبه الـ AM في الباور والـ BW والـ Spectrum إلا أن الـ NBFM أفضل لأن الـ NBFM له مقاومة عالية للـ noise ولذلك الـ NBFM على الـ AM .

(b) for W.B.F.M

① $\beta \geqslant 1$ → بالتالي لن نستطيع إختصار $\cos(\beta \sin(\,))$
و $\sin(\beta \sin(\,))$

كما في ال Narrow Band فهتختلف المعادلة هنا

$$S(t)_{WBFM} = \sum_{n=-\infty}^{\infty} \underbrace{J_n(\beta)}_{\text{Bessel's fn}} A_c \cos(2\pi(f_c + n f_m)t)$$ مهم

$$\therefore S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta) \left[\delta(f - (f_c + n f_m)) + \delta(f + (f_c + n f_m)) \right]$$ مهم

- B.W. (Carson's Rule) $= 2(\beta + 1) f_m$ مهم $= 2(\Delta f + f_m)$

* $P = \frac{A_c^2}{2} \sum_{n=-\infty}^{\infty} J_n^2(\beta)$. which is $\frac{V_p^2}{2}$ of the Cosine signal

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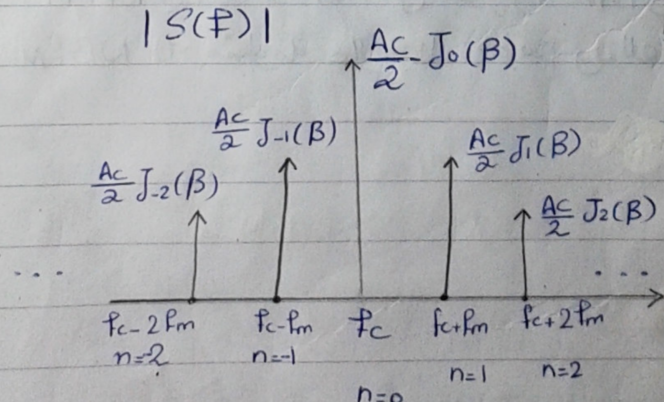
* Note :-

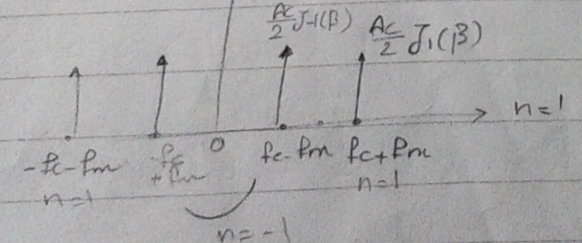


at n=1

$\frac{A_c}{2} J_1(\beta) [\delta(f - (f_c + f_m)) + \delta(f + (f_c + f_m))]$

n=-1.

$\frac{A_c}{2} J_{-1}(\beta) \cdot [\delta(f - (f_c - f_m)) + \delta(f + (f_c - f_m))]$



B.W is extended to infinity (Wide Band) but Carson's rule only consider B.W $= 2(\beta + 1) f_m$ and as it shows the high components has low amplitude that can be ignored.

In FM Questions

① Find the modulation index (β)

Given: f_m الصورة العامة لمعادلة
و استخرج منها ال β

Given: f_m , Δf

$$\beta = \frac{\Delta f}{f_m}$$

② NBFM or WBFM ??

$0 < \beta < 1$

NBFM

لو طلب

- $B.W. = 2 f_m$

$\beta \geq 1$

WBFM

لو طلب

- Carson's rule

$B.W. = 2(\beta+1) \cdot f_m$

- $$Power = P_c + P_{DSB}$$
$$= \frac{A_c^2}{2} + \frac{A_c^2 \beta^2}{4}$$
$$= \frac{A_c^2}{2}\left(1 + \frac{\beta^2}{2}\right)$$

$$P_T = P_c\left(1 + \frac{\beta^2}{2}\right)$$

نفس الباور لل DSBTC بس
بدل ال $\mu \leftarrow \beta$

- $$Power = \frac{A_c^2}{2} \sum_{n=-\infty}^{\infty} J_n^2(\beta)$$

نعرفها من جدول Bessel

لو طلب ال Spectrum
عدد ال Sidebands حول ال f_c

$$n = \frac{B.W}{2 f_m}$$

ونحسب الباور على هذا الجزء فقط

* في الحالتين التردد اللحظي $f_i(t) = f_c + K_f \cdot m(t)$ *

FM Sheet

Before answering any FM question determine from β is it a NBFM or WBFM ?!

① $\Delta f = 8\ kHz$ $\quad f_m = 4\ kHz$ $\quad$ BW of fm = ??

$A_c = 18\ V$ $\quad f_c = 3\ MHz$ $\quad$ FM eqn = ??

B.W. of fm

NBFM	WBFM
$2f_m$	$2(\beta+1)f_m$
NBFM	WBFM
$0 < \beta < 1$	$\beta \geq 1$

- $\beta = \dfrac{\Delta f}{f_m} = \dfrac{8}{4} = 2 > 1 \longrightarrow$ WBFM

- BW of FM signal $= 2(\beta+1) f_m$
$= 2(2+1)(4\ KHz)$
$= 24\ KHz$

$$S(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$$

- $S(t) = 18 \cos(2\pi \cdot 3*10^6 t + 2 \sin(2\pi \cdot 4*10^3 t))$

Note

$J_{-n}(\beta) = J_n(\beta)$ $\quad$ n even

$J_{-n}(\beta) = -J_n(\beta)$ $\quad$ n odd

$J_{-n}(\beta) = (-1)^n J_n(\beta)$

$f_i(t) = f_c + K_f . A_m$

Hz, Hz, $\frac{Hz}{V}$, V

② $A_m = 4V$ $f_m = 1200$ Hz $\omega = 2\pi f$

$A_c = 8V$ $f_c = 4$ MHz ↳ rad/s ↳ Hz or s^{-1}

$K_f = 5652$ rad/s/volt $\div 2\pi = 899.54$ Hz/s

$S(t) = ?$ $\Delta f = ?$ $B.W. = ?$

Sol.

$\beta = \frac{\Delta f}{f_m}$, $\cdot\ \Delta f = K_f . A_m = 899.5 * 4 = 3598$ Hz

$\beta = \frac{899.5 * 4}{1200} \cong 3 > 1 \longrightarrow \therefore$ WBFM

$\cdot$ $B.W. = 2(\beta + 1) f_m$

$= 2(3 + 1).(1200)$

$= 9600$ KHz

$S(t) = A_c \cos(2\pi f_c t + \beta \sin 2\pi f_m t)$

$= 8 \cos(2\pi * 4 * 10^6 t + 3 \sin(2\pi * 1200 t))$

---

③ $S(t) = 10 \cos(2\pi(4 * 10^6)t + 0.8 \sin(2\pi . 600 t))$ volts

$A_m = 4$ V

a) Modulation Type = ? B.W. = ? $P_e = ?$

$S(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$

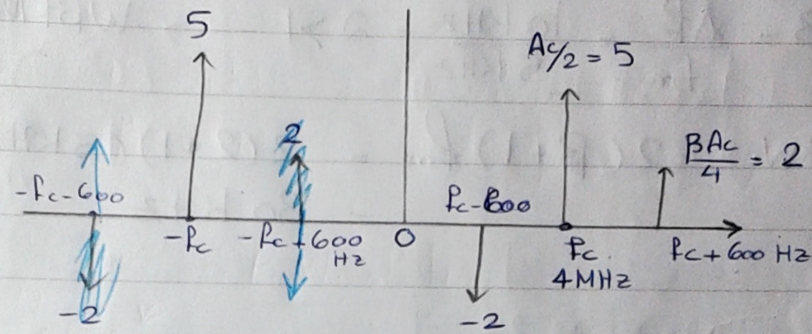
$\therefore A_c = 10$ V, $f_c = 4 * 10^6$ Hz, $f_m = 600$ Hz

$\beta = 0.8 < 1 \longrightarrow$ NBFM

$B.W. = 2 f_m = 2 * 600 = 1200$ Hz $= 1.2$ KHz

$$P_c = \frac{A_c^2}{2} = \frac{10^2}{2} = 50 \text{ watts}$$

b) Spectrum = ? $\quad \Delta f = ?$



$$\beta = \frac{\Delta f}{f_m} \longrightarrow \Delta f = \beta \cdot f_m = 0.8 * 600 = 480 \text{ Hz}$$

if asked about K_f : $K_f = \frac{\Delta f}{A_m} = \frac{480}{4} = 120$ Hz/volt

c)

$$A_m \xrightarrow{\text{Changed}} A_m'$$

$4V \to 7V$

$f_m = 600 \to f_m' = 350$

$$\beta' = \frac{K_f \cdot A_m'}{f_m'} = \frac{120 * 7}{350} = 2.4 > 1 \longrightarrow \text{WBFM}$$

إتحول النظام

ل WBFM بالتالي ال B.W زاد

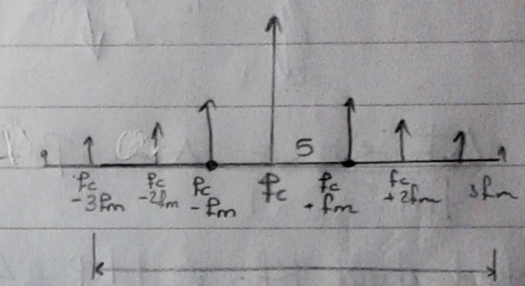
$$BW' = 2(\beta' + 1) f_m'$$
$$= 2(1.4 + 1)(350)$$
$$= 1.68 \text{ KHz}.$$

(4) $\Delta f = 10\ KHz$ $f_m = 5\ KHz$

$A_c = 10\ V$ $f_c = 500\ KHz$

$$\beta = \frac{\Delta f}{f_m} = \frac{10\ KHz}{5\ KHz} = 2 > 1 \longrightarrow WBFM$$

$$B.W. = 2(\beta+1) f_m = 2(2+1)(5\ KHz)$$
$$= 30\ KHz$$

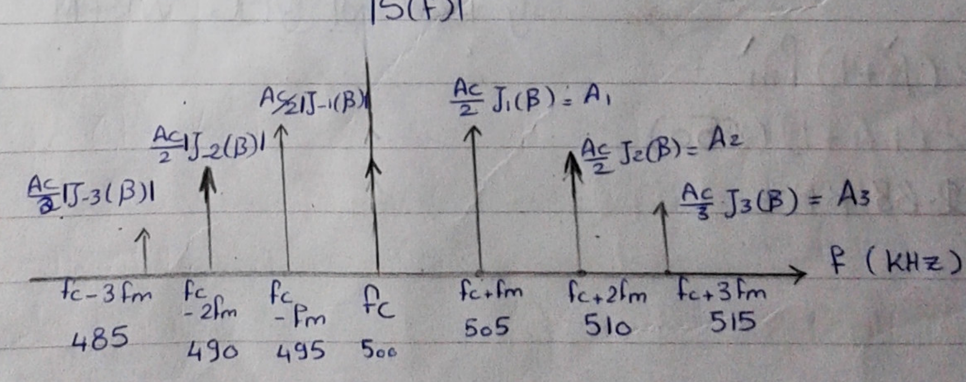


a) No. of sets of side frequencies $= \dfrac{30\ KHz}{2\ f_m}$

$$\frac{B.W}{2 f_m} = 3$$

$$\boxed{n = 3}$$

b) Spectrum ??



$$A_0 = \frac{A_c}{2} J_0(2) = 5 * 0.2239 = 1.1195$$

$A_1 = 5 * J_1(2) = 5 * 0.5767 = 2.8835$

$A_2 = 5 * J_2(2) = 5 * 0.3528 = 1.764$

$A_3 = 5 * J_3(2) = 5 * 0.1289 = 0.6445.$

c) Power

$$P = \frac{A_c^2}{2} \sum_{n=-\infty}^{\infty} J_n^2(\beta) = \frac{A_c^2}{2}\left[J_0^2(\beta) + J_1^2(\beta) + J_{-1}^2(\beta) + \cdots \right]$$

$$= \frac{A_c^2}{2}\left[J_0^2[\beta] + 2J_1^2(\beta) + 2J_2^2(\beta) + 2J_3^2(\beta)\right]$$

$$= 2\sum_{n=-\infty}^{\infty} A_i^2 = 2(A_0^2 + 2A_1^2 + 2A_2^2 + 2A_3^2) + \cdots$$

$$= 2A_0^2 + 4A_1^2 + 4A_2^2 + 4A_3^2 \quad \text{until } A_n \ (n=3 \text{ here})$$

$$= 2(1.1195^2 + 2 * 2.8835^2 + 2 * 1.764^2 + 2 * 0.6445^2)$$

$$= 2(1.1195)^2 + 4(2.8835)^2 \qquad A_{-1} = -A_1$$

$$+ 4(1.764)^2 + 4(0.6445)^2 \qquad \therefore A_{-1}^2 = A_1^2$$

$= 49.87$ watts

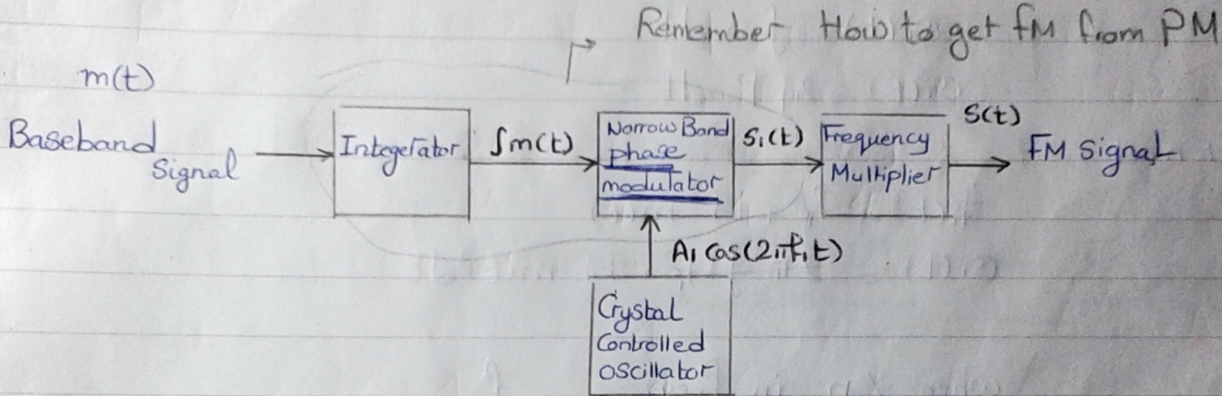
$$\frac{A_c^2}{2} J_1^2(\beta) \quad \frac{A_c^2}{2} J_1(\beta)$$

A^2

Generation of FM Waves :-

- There are two methods of FM waves generation :
  1. Indirect Method
  2. Direct Method

① Indirect FM



This modulator Produces WBFM signal from NBFM signal that's why it is called indirect.

$$S_1(t) \longrightarrow \text{NBFM}$$

It is then multiplied in Frequency by factor n to Produce the desired WBFM.

$$S_1(t) = A_1 \cos(2\pi f_1 t + 2\pi K_f \int m(t)\,dt)$$

$$S_1(t) = A_1 \cos(2\pi f_1 t + \underbrace{\beta_1}_{\frac{k_f \cdot A_m}{f_m} = \frac{\Delta f}{f_m}} \sin(2\pi f_m t))$$

For m(t) casine signal

- β_1 is Kept small $\simeq 0.5$ to keep the distortion minimum in the medulator

$$f_1 \longrightarrow \boxed{\times n} \longrightarrow n f_1 = f_c$$

$$\beta_1 \longrightarrow \boxed{\times n} \longrightarrow n \beta_1 = \beta$$

$$S_1(t) = A_1 \cos(2\pi n f_1 t + 2\pi n K_f \int m(t)\,dt)$$

$$= A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t)).$$

where $f_c = n f_1$

$\beta = n \beta_1$

So, by properly choosing the multiple n, we can determine the modulation index β and f_c as desired.

توضيح :-

$\theta_i(t) = \int \omega \, dt$

$\theta_i(t) = 2\pi \int f \cdot dt$

After mod.

$f_i(t) = f_1 + k_f \cdot m(t)$

$\theta_i(t) = 2\pi \int (f_1 + k_f \cdot m(t)) dt$

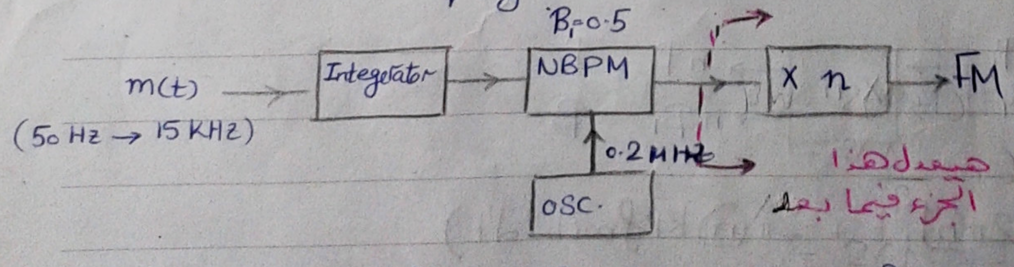
when $\times n$ in freq. $\quad f$

$= 2\pi \cdot n \cdot \int \underbrace{f_1 + k_f \cdot m(t)} dt$

$\therefore \theta_i(t) = 2\pi \boxed{n} f_1 t + 2\pi \boxed{n} k_f \int m(t) \, dt$

- This method can be unpractical & need more enhancing in practical systems ...

For example, it is required to send FM wave that has $f_c = 90$ MHz and maximum frequency deviation $\Delta f = 75$ kHz.



- the maximum β for NBPM to operate satisfactory is 0.5, the m(t) signal

has frequencies from 50 Hz to 15 KHz.

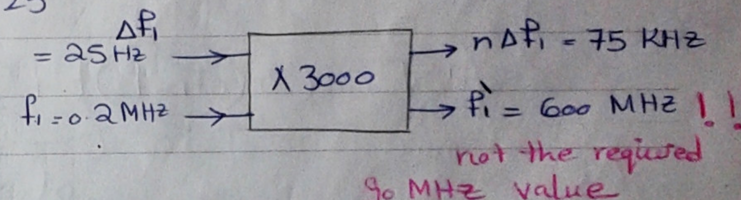
So, for least f_m, β must be 0.5 as β is inversely Proportional to f_m.

$$\beta = \frac{\Delta f}{f_m}$$

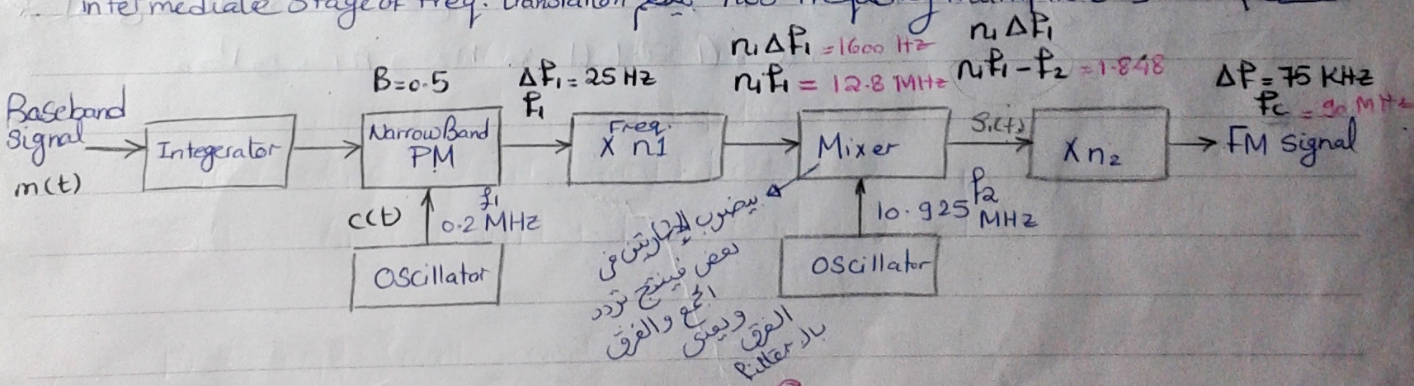
$$0.5 = \frac{\Delta f_1}{50} \longrightarrow \Delta f_1 = 0.5 \times 50 = 25 \text{ Hz}$$

Therefore, for maximum $\Delta f = 75$ KHz

$$n = \frac{75 \text{ KHz}}{25} = 3000$$



• يبقى الهدف دلوقت إني أحقق المواصفات كلها Δf وكمان f_c لذلك عمل النظام ده لـ Two frequency multipliers بينهم intermediate stage of freq. translation.



$$n_1 n_2 = 3000 \rightarrow 1$$

So, $\Delta f_1 = 25$ Hz $\longrightarrow$ $\Delta f = 75$ KHz

$$n_1 f_1 - f_2 = \frac{f_c}{n_2}$$

$$0.2 n_1 - 10.925 = \frac{90}{n_2} \rightarrow 2$$

$$n_1 = 64.3 \simeq 64$$

$$n_2 = 46.7 \simeq 48$$

64 & 48 can be factorized by multiples of 2 & 3 which means that multiplication Process can be done by X2 and X3 freq. multipliers.

$$\begin{array}{r|l} 3 & 48 \\ 2 & 16 \\ 2 & 8 \\ 2 & 4 \\ 2 & 2 \\ & 1 \end{array} \qquad \begin{array}{r|l} 2 & 64 \\ 2 & 32 \\ 2 & 16 \\ 2 & 8 \\ 2 & 4 \\ 2 & 2 \\ & 1 \end{array}$$

$$S_1(t) = A_1 \cos(2\pi n f_1 t + B_1 \sin(2\pi f_m t)) * \cos(2\pi f_2 t)$$

$$= \frac{A_1}{2}\left[\cos(2\pi (n f_1 - f_2) t + \underset{\downarrow}{B_1} \sin(2\pi f_m t)\right] + \cos(\text{المجموع})$$

نری ما هي

* لو في مسألة أداك معطى $f_1, f_2, f_c, \Delta f$ وطلب تعمل design يبقى I
مطلوب n_1 و n_2 من المعادلة ① و ②

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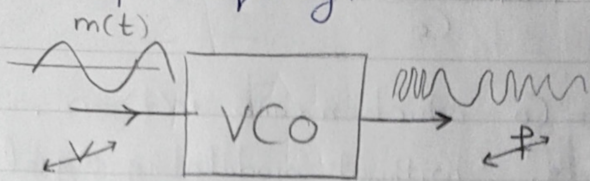
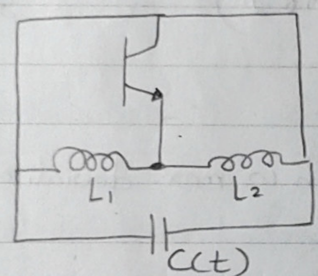
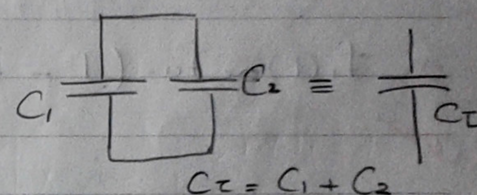
② Direct FM

The idea of VCO "Voltage-Controlled Oscillator" which its input voltage changes the o/p frequency.



One way to implement the VCO is by Hartley oscillator which has fixed capacitor shunted by variable capacitor which its capacitance changes with i/p voltage.





$C_T = C_1 + C_2$

C(t): Total Capacitance (Fixed + Variable)
↳ changes with i/p voltage

The freq. of oscillation of Hartley

عوض

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)\,C(t)}}$$

التردد عند لحظة معتمد على C عند هذه اللحظة

$$C(t) = C_0 + \Delta C \cos(2\pi f_m t)$$

و C(t) عند لحظة تعتمد على الـ m(t) عند هذه اللحظة

المتغير

↳ assume $m(t) \Rightarrow \cos(2\pi f_m t)$

max. change at the variable C when m(t)=max.

$$f_i(t) = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{(L_1+L_2)C_0 + (L_1+L_2)\Delta C \cos(2\pi f_m t)}}$$

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)C_0\left(1+\frac{\Delta C}{C_0}\cos(2\pi f_m t)\right)}}$$

$$f_i(t) = f_o \cdot \left(1 + \frac{\Delta C}{C_0}\cos(2\pi f_m t)\right)^{-1/2}$$

$f_o \rightarrow$ freq. at $C(t) = C_0$ which means $m(t)=0$.
↳ unmodulated freq. "without modulating signal $m(t)$"

$$(1+x)^n \simeq 1 + nx + \underbrace{\frac{n(n-1)}{2!}x^2 + \dots}$$

↳ v. small terms at ΔC small so, ignore them here

$$f_i(t) \simeq f_o \cdot \left(1 - \frac{\Delta C}{2C_0}\cos(2\pi f_m t)\right)$$

مهم

$\Delta C \rightarrow$ max. value of var. C which is max. deviation from C_0
$C_0 \rightarrow$ Causes f_o "$m(t)=0$"

$$\therefore \frac{\Delta C}{2C_0} = -\frac{\Delta f}{f_o}$$

f_c $\qquad K_f \cdot m(t) = \underbrace{K_f A_m}_{\Delta f} \cos(2\pi f_m t)$

$$f_i(t) \simeq f_o + \Delta f \cos(2\pi f_m t)$$

which is the instantenous freq. of an FM wave.

$$\Delta f = \frac{1}{2\pi\sqrt{L_T \cdot \Delta C}} \qquad \sqrt{\frac{\Delta C}{C_0}}^{-1/2} = \frac{\Delta f}{f_o}$$

$$f_o = \frac{1}{2\pi\sqrt{L_T \cdot C_0}} \qquad 1 - \frac{\Delta C}{2C_0} = \frac{\Delta f}{f_o}$$

Demodulation of FM waves :-

- The process that enables us to recover the original modulating wave from the frequency modulated wave.

- There are two methods for FM demodulation:
  1. Frequency discriminator
  2. Phase-Locked Loop.

- In both cases, the objective is to produce a transfer charachteristic which is the inverse of that of the frequency modulator.

① Frequency Discriminator

- It consists of a slope circuit followed by an envelope detector.
- An ideal slope circuit is charachterized by a transfer function that is purely imaginary, varying linearly with the frequency inside the prescribed frequency interval.

$$H_1(f) = \begin{cases} j2\pi a\left(f - f_c + \frac{B_T}{2}\right) & f_c - \frac{B_T}{2} < f \le f_c + \frac{B_T}{2} \quad \rightarrow ① \\ j2\pi a\left(f + f_c - \frac{B_T}{2}\right) & -f_c - \frac{B_T}{2} < f < -f_c + \frac{B_T}{2} \\ 0 & \text{elsewhere} \end{cases}$$

$= \tilde{H}_1(f - f_c)$
$f = f + f_c$
Transfer fn. of slope circuit

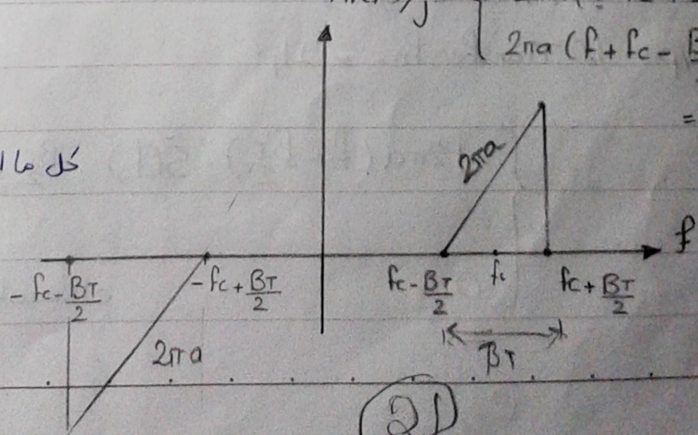
to get $\tilde{H}_1(f)$

$$y = mx + c = \underbrace{2\pi a}_{m} f - 2\pi a\left(f_c + \frac{B_T}{2}\right)$$

$$H_1(f)/j = \begin{cases} 2\pi a\left(f - f_c + \frac{B_T}{2}\right) & f_c - \frac{B_T}{2} < f < f_c + \frac{B_T}{2} \\ 2\pi a\left(f + f_c - \frac{B_T}{2}\right) & -f_c - \frac{B_T}{2} < f < -f_c + \frac{B_T}{2} \end{cases}$$

$$= 2\pi a f + 2\pi a\left(f_c - \frac{B_T}{2}\right)$$

كل ما ال f تزيد الجهد بيزيد



$$s(t) \xrightarrow{\text{FM wave}} \boxed{\text{Slope cct.}\ H_1(f)} \longrightarrow s_1(t)$$

$B.W. = B_T$

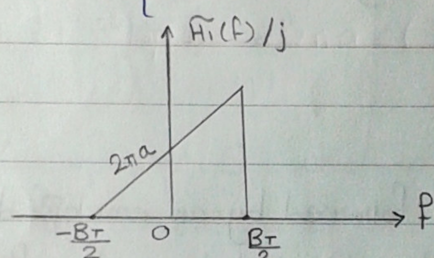
f_c

$\tilde{H}_1(f)$: Complex transfer fn. of slope circuit

$$\tilde{H}_1(f-f_c) = H_1(f) \quad f>0$$

$\therefore$ From (1)

$$\tilde{H}_1(f) = \begin{cases} j\,2\pi a\left(f+\frac{B_T}{2}\right) & -\frac{B_T}{2} \leq f \leq \frac{B_T}{2} \\ 0 & \text{elsewhere} \end{cases}$$



$s(t)$: the incoming FM wave

$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right)$$

$\tilde{s}(t)$: the complex envelope of FM wave.

أسهل في التفاضل $\quad \tilde{s}(t) = A_c \exp\left(j \cancel{2\pi f_c t}\ 2\pi k_f \int_0^t m(t)\,dt\right) \rightarrow (2)$

$$\tilde{S}_1(f) = \tilde{H}_1(f) \cdot \tilde{S}(f)$$

F.T. of $\tilde{s}(t)$ $\swarrow$

o/p = Transfer fn. I/p

$$= \begin{cases} j\,2\pi a\left(f+\frac{B_T}{2}\right) \cdot \tilde{S}(f) & -\frac{B_T}{2} \leq f \leq \frac{B_T}{2} \\ 0 & \text{elsewhere} \end{cases}$$

$$\tilde{S}_1(f) = j2\pi f a \cdot \tilde{S}(f) + j\pi a B_T \tilde{S}(f)$$

From F.T. Properties " differentiation "

$$\frac{d\,g(t)}{dt} \rightleftharpoons G(f)\,(j2\pi f)$$

$$\therefore \tilde{S}_1(t) = a \cdot \frac{d\tilde{S}(t)}{dt} + j\pi a \cdot B_T \cdot \tilde{S}(t)$$

$$\tilde{S}_1(t) = a \cdot \left[\frac{d\tilde{S}(t)}{dt} + j\pi B_T \tilde{S}(t)\right]$$

from ② in ①   عوض عن $\tilde{S}(t)$

$$\tilde{S}_1(t) = a \cdot \Big[A_c\,(j2\pi k_f\, m(t))\,\exp\!\big(j2\pi k_f \textstyle\int m(t)\,dt\big) + j\pi B_T \cdot A_c \cdot \exp\!\big(j2\pi k_f \textstyle\int m(t)\,dt\big)\Big]$$

$$= j\pi a \cdot A_c \cdot B_T \left(\frac{2k_f m(t)}{B_T} + 1\right) \cdot \exp\!\big(j2\pi k_f \textstyle\int m(t)\,dt\big)$$

$\therefore$ The desired response of the slope circuit

هرجع $H_1(f)$ عند ال f_c

$$S_1(t) = \operatorname{Re}\left[\tilde{S}_1(t) \cdot \exp(j2\pi f_c t)\right]$$

$$S_1(t) = \operatorname{Re}\left[j\pi a \cdot A_c B_T \left(\frac{2k_f m(t)}{B_T} + 1\right) \cdot \exp\!\Big(j2\pi\big(f_c + k_f \textstyle\int_0^t m(t)\big)\,t\Big)\right]$$

$$j = 1\angle 90^\circ \quad \therefore \exp\left(\cdots + \frac{\pi}{2}\right)$$

$$= \underbrace{\pi \cdot a A_c B_T \left(\frac{2k_f m(t)}{B_T} + 1\right)}_{AM} \cos\Big(2\pi\big(f_c + \underbrace{k_f \textstyle\int m(t)\,dt}_{FM}\big) + \frac{\pi}{2}\Big)$$

The o/p $S_1(t)$ is a hybrid-modulated wave that has both AM & FM Carrier wave. " the amp. & the Freq. of the Cosine Carrier both vary with $m(t)$ ".

So, to extract $m(t)$ we will use envelope detector circuit that passes the signal envelope. "الـ term اللي جنب الـ cos"

$$|S_1(t)| = \underbrace{\pi \cdot B_T \cdot a\, A_c}_{\Downarrow}\left(1 + \frac{2K_f}{B_T}\, m(t)\right).$$

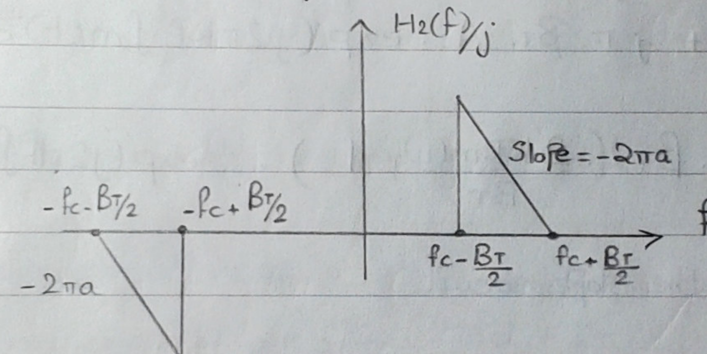
(خفيفة dc مجموعة على $m(t)$) Bias term

للحصول على $m(t)$ من غير هذه الكمية

Bias term $= \pi \cdot B_T \cdot a \cdot A_c$

↳ Constant that determines the slope of $\frac{H_1(f)}{j}$

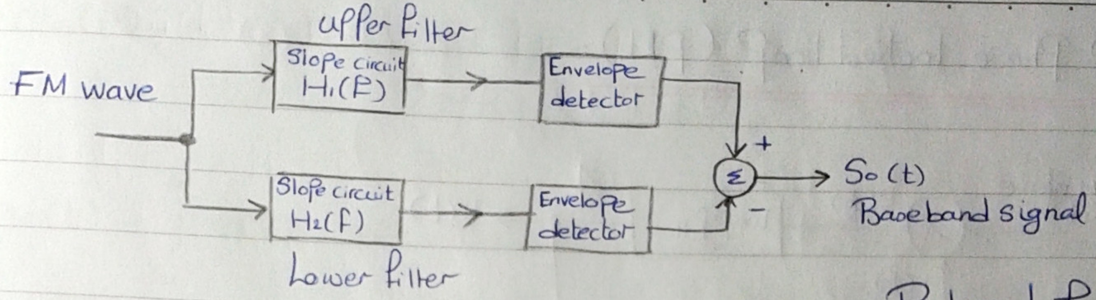
This bias term can be removed by subtracting $S_2(t)$ from $S_1(t)$
$S_2(t)$ has $H_2(f)$ of slope $= -2\pi a$ (has negative a)



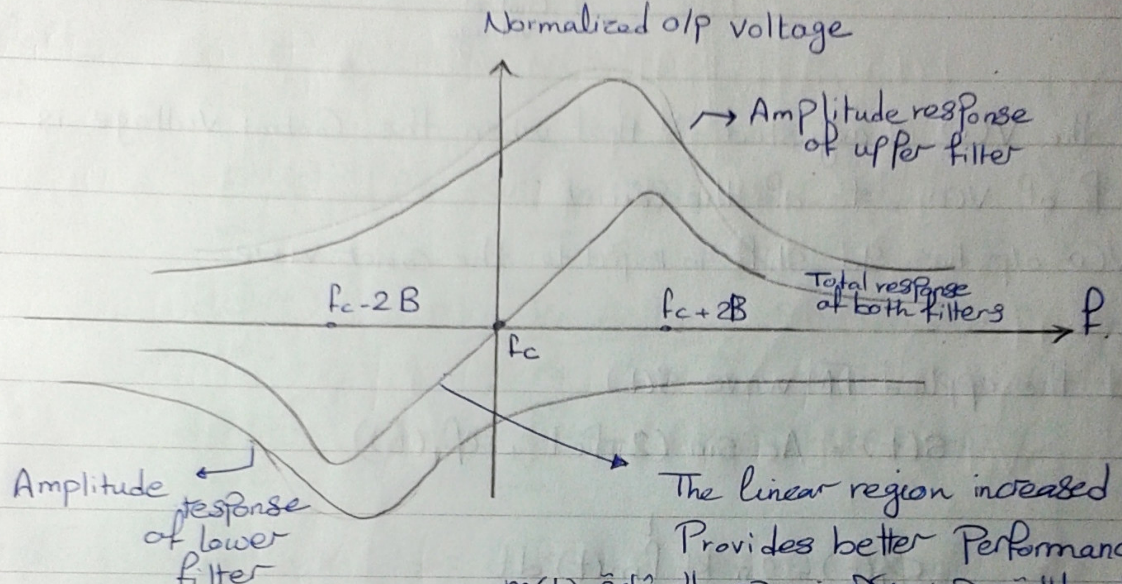
$\therefore\ S_o(t) = |S_1(t)| - |S_2(t)|$

$$|S_2(t)| = \pi B_T a A_c \left(1 - \frac{2K_f}{B_T}\, m(t)\right)$$

$\therefore\ \boxed{S_o(t) = 4\pi K_f a A_c\, m(t)}$ → وهو المطلوب



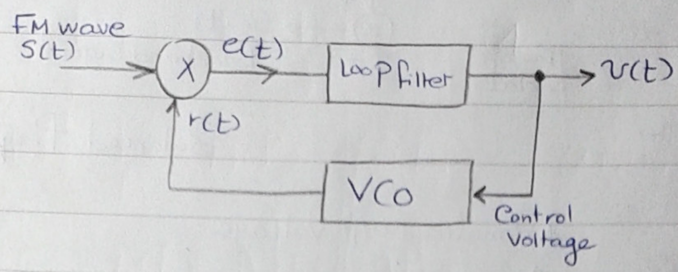
Balanced freq. discriminator



The linear region increased which Provides better Performance

لكن كل ما التردد هيزيد هيزيد جهد الاشارة m(t) بشكل خطي و هنا هو عكس ال FM في العمل
"demodulation"

② Phase-locked Loop (PLL) :-



- Assume that the VCO is adjusted so that when the Control Voltage is zero: ① f of VCO = f_c of the carrier

  ② VCo o/p has 90° shift in respect to the carier wave.

- Suppose that the applied FM wave $S(t)$

$$S(t) = A_c \sin(2\pi f_c t + \phi_1(t))$$

$$\phi_1(t) = 2\pi\, k_f \int_0^t m(t)\, dt$$

VCO o/p $\leftarrow$ $r(t) = A_v \cos(2\pi f_c t + \phi_2(t))$

freq. depend on I/p voltage

Same FM eqn

$$\phi_2(t) = 2\pi k_v \cdot \int_0^t v(t)\, dt$$

that's why

a VCO can be used as

FM modulator

when multiplying $S(t) \times r(t)$

$K_m \cdot A_c A_v \cdot \sin(4\pi f_c t + \phi_1(t) + \phi_2(t)) \rightarrow$ High freq. Component

$+ K_m A_c A_v \sin(\phi_e(t)) \rightarrow$ Low-freq. Component

$$e(t) = K_m \cdot A_c \cdot A_v \sin(\phi_e(t))$$

$$\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)].$$

$$\phi_e(t) = \phi_1(t) - \phi_2(t)$$
$$= \phi_1(t) - 2\pi K_v \int_0^t v(t)\,dt$$

$$v(t) = \int_{-\infty}^{\infty} e(\tau) \cdot h(t-\tau)\, d\tau \longrightarrow$$ Frequency تناظر في ال

$$V(f) = E(f) \cdot \underset{\downarrow}{H(f)}$$
of Loop filter

$$\frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - 2\pi K_v \cdot v(t)$$

$$= \frac{d\phi_1(t)}{dt} - 2\pi \cdot K_v \cdot \int_{-\infty}^{\infty} e(\tau) \cdot h(t-\tau)\, d\tau$$

$$= \frac{d\phi_1(t)}{dt} - 2\pi\, \underbrace{K_v \cdot K_m A_c A_v}_{K_o} \int_{-\infty}^{\infty} \sin(\phi_e(\tau)) \cdot h(t-\tau)\, d\tau$$

$$= \frac{d\phi_1(t)}{dt} - 2\pi K_o \int_{-\infty}^{\infty} \sin(\phi_e(\tau))\, h(t-\tau)\, d\tau$$

when ($\phi_e = 0$) the locked loop is said to be imphase-lock and the VCO generates its f_c frequency

$$S(t) = A_c \cos(2\pi f_c t + \phi)$$
$$r(t) = A_c \sin(2\pi f_c t + \phi)$$

S, r are 90° phase shifted

$\phi_e = 0$ VCO (f_c)

$\sin[\phi_e(t)] \simeq \phi_e(t)$ for small values

$$\frac{d\phi_e(t)}{dt} + 2\pi k_o \int_{-\infty}^{\infty} \phi_e(\tau)\, h(t-\tau)\, d\tau = \frac{d\phi_1(t)}{dt}$$

$\Downarrow$ F.T.

$$\phi_E(f) \cdot (j\cancel{2\pi} f) + \cancel{2\pi} K_o \cdot \phi_E(f) \cdot H(f) = \phi_1(f) \cdot (j\cancel{2\pi} f)$$

$$\phi_E(f)\,(jf + K_o H(f)) = \phi_1(f) \cdot f$$

$$\phi_E(f) = \phi_1(f) \cdot \cancel{f} \Big/ j\cancel{f} + \frac{K_o \cdot H(f)}{f}$$

$$\phi_e(f) = \frac{1}{1 + L(f)} \cdot \phi_i(f) \quad ③$$

$$L(f) = k_o \cdot \frac{H(f)}{jf} \quad ④$$

$$V(f) = \frac{k_o}{k_v} H(f) \cdot \phi_e(f)$$ عوض من ④

$$V(f) = L(f) \cdot \phi_e(f) \cdot \frac{jf}{k_v}$$ عوض من ③

$$V(f) = \frac{jf}{k_v} \cdot \frac{L(f)}{1 + L(f)} \phi_i(f)$$

If $|L(f)| \gg 1$

$$V(f) \simeq \frac{jf}{k_v} \phi_i(f)$$

$$\Big\downarrow \text{ I.F.T.}$$

$$v(t) \simeq \frac{1}{2\pi k_v} \cdot \frac{d\phi_i(t)}{dt}$$

$$\phi_i(t) = 2\pi k_f \int m(t)\, dt \longrightarrow \frac{d\phi_i(t)}{dt} = 2\pi k_f \cdot m(t)$$

$$\therefore \boxed{v(t) \simeq \frac{k_f}{k_v} \cdot m(t)} \rightarrow$$ وهو المطلوب